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Requiem for the Born–Infeld Electron

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It is shown that the supposed existence of a particlelike solution in the Born–Infeld electrodynamics is due to an error, and comments are made on the significance of this result for unitary and unified theories.

It is surprising that a fundamental error could persist for many years without detection in a well-studied theory, but this apparently was the case with the nonlinear electrodynamics of Born and Infeld (1934). This theory enjoyed considerable popularity in the 1930s and 1940s, in part because it was believed to provide a unitary model of a charged particle as a region of very high, but finite, electromagnetic field strength.

For static, spherically symmetric electric fields, the field equations of the theory reduce to

$$E = -dV/dr \tag{1}$$

and

$$\nabla\cdot\mathbf{D} = r^{-2}d(r^2D)/dr = 0 \tag{2}$$

Here the displacement $\mathbf{D}$ is given by $\mathbf{D} = (1 - E^2/b^2)^{-1/2}\mathbf{E}$, with b the critical field strength of the theory. From equation (2), Born and Infeld concluded that $D = e/r^2$, with e a constant, gave the desired solution. Though D then becomes infinite at $r = 0$, the field strength E remains finite. (In fact, $E(0) = b$.) The potential V can then be found from (1) as an elliptic integral. The energy associated with this field can be evaluated and turns out to have a finite value, $(2/3)(e^3b)^{1/2}K(2^{-1/2})$, where K denotes

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the complete elliptic integral of the first kind. This result seems a pleasant contrast to the infinite result encountered for a point charge in the Maxwell–Lorentz theory.

However, Deser (1976) has shown that no such particlelike solutions exist for a large class of theories, and has demonstrated this explicitly for the Born–Infeld theory. How then were Born and Infeld and many others (including, it must be confessed, the present author) deceived? The fact is that $\mathbf{D} = e\mathbf{r}/r^3$ is *not* a solution of equation (2) at all, but of

$$\nabla \cdot \mathbf{D} = 4\pi e \delta(\mathbf{r}) \tag{3}$$

(The standard procedure for showing this involves integration over a small sphere surrounding the origin and use of Gauss' theorem.) (It is worthy of note that at a critical point Born and Infeld do not write the relevant field equation as $r^{-2}d(r^2D)/dr = 0$ but simply as $d(r^2D)/dr = 0$. [See their equation (6.3).] This is a quite different thing when one has to be concerned with the behavior at $r = 0$.)

In other words, the result of Born and Infeld is not a solution of their homogeneous field equations at all, but of a set of equations in which an externally prescribed charge density has been included. The procedure is exactly that which is routinely followed in order to obtain Coulomb's law from Maxwell's equations. There, however, one recognizes that the Coulomb potential is not a solution of the homogeneous Laplace equation everywhere, but of the Poisson equation with a δ-function charge density. There is no problem with this since the Maxwell theory does not purport to be a unitary theory of charges and fields.

A goal of the Born–Infeld theory was to have a unitary theory in which charged particles and electromagnetic fields would be represented as different aspects of the same fundamental entity. But since foreign charges have to be introduced, the goal is not attained. Any foreign charge distribution can be built up as a superposition of δ-function charges, so the theory is incomplete. (See also Wheeler, 1961.) The fact that the solution of the revised equations has finite energy is irrelevant to the question of the unitary character of the theory.

Born and Infeld expressly distinguished between their sense of a *unitary* theory and the idea of a unified field theory in which, for example, gravitational and electromagnetic fields would be given a unified description. However, it is interesting that the Born–Infeld theory has mathematical features in common with nonsymmetric unified field theories of the Einstein–Schrödinger type (Murphy, 1975). The negative result discussed here may thus justify some scepticism about the ability of such theories to represent particles in a nonsingular manner, even at the classical level.

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